Color Algebras

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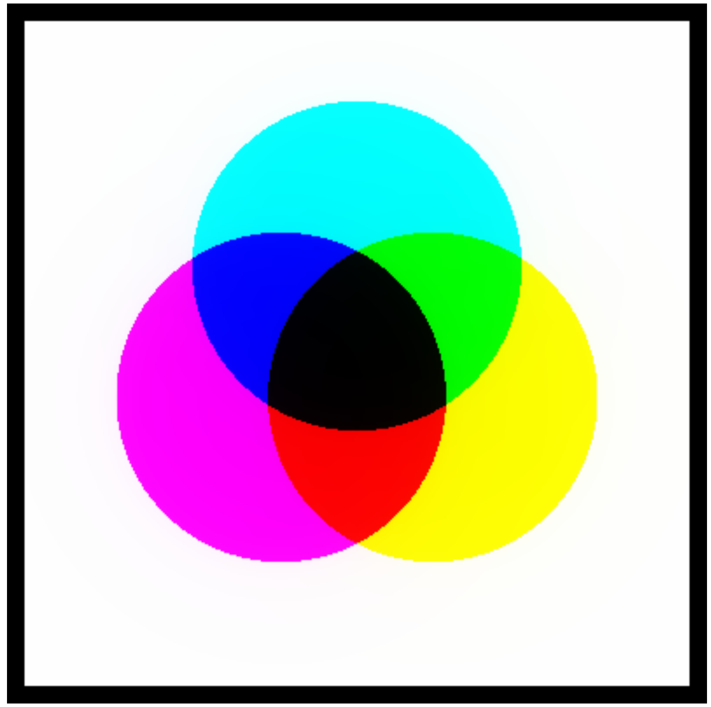
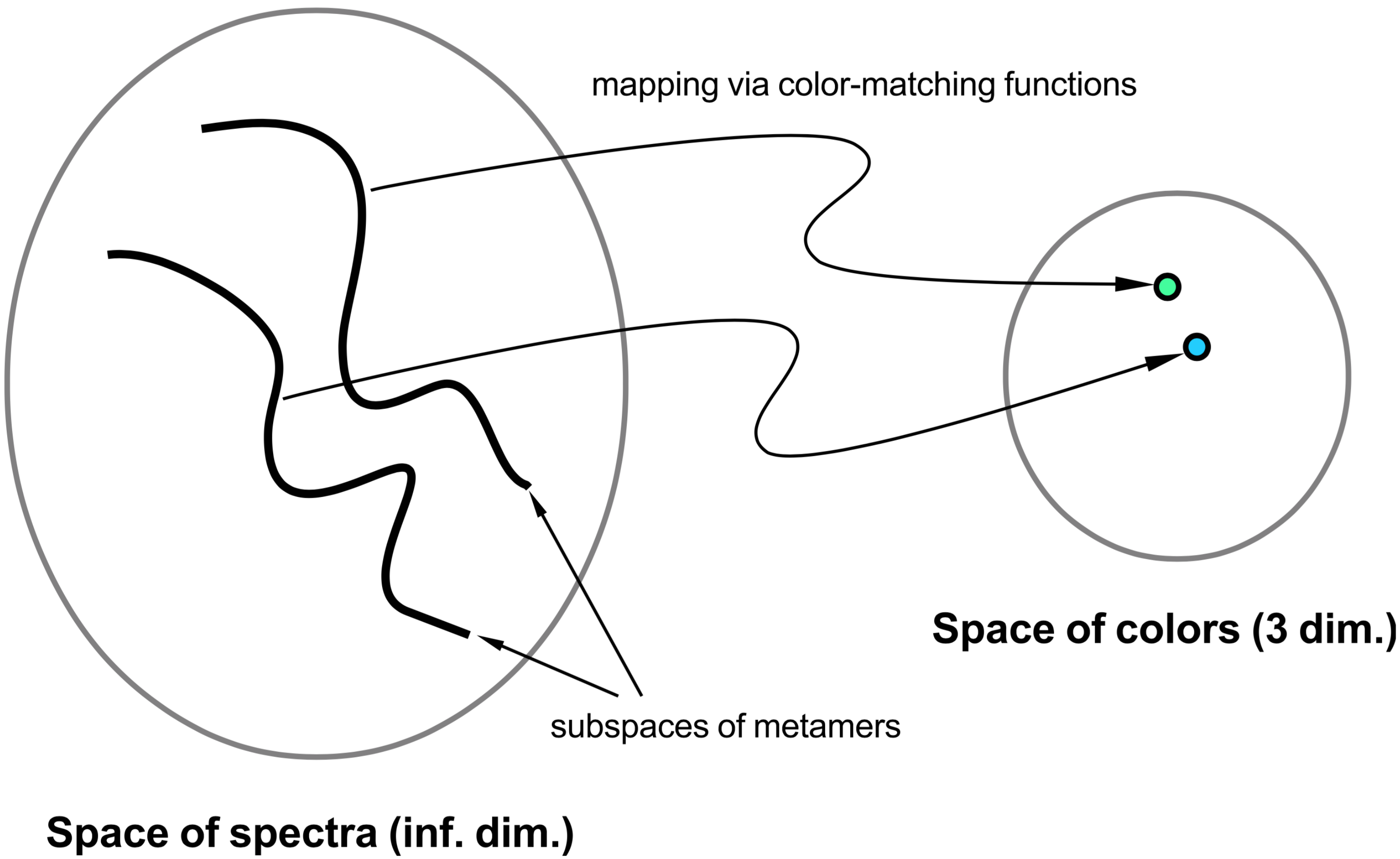
Two kinds of color mixture

Additive



Subtractive (multiplicative)

Metameric spectra



Color algebra

- Additive color mixture perfectly described

  by vector addition of colors

$$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a},$$

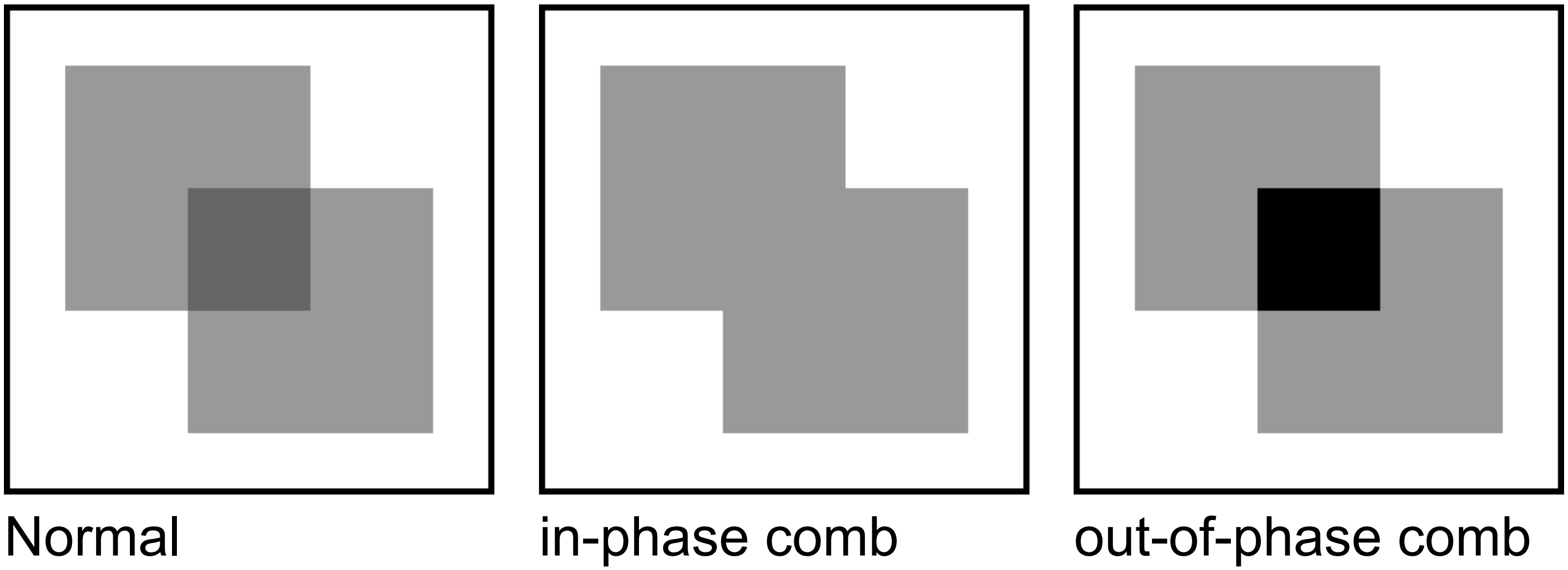
$$(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$$

- Subtractive color mixture NOT well-defined from input colors, so

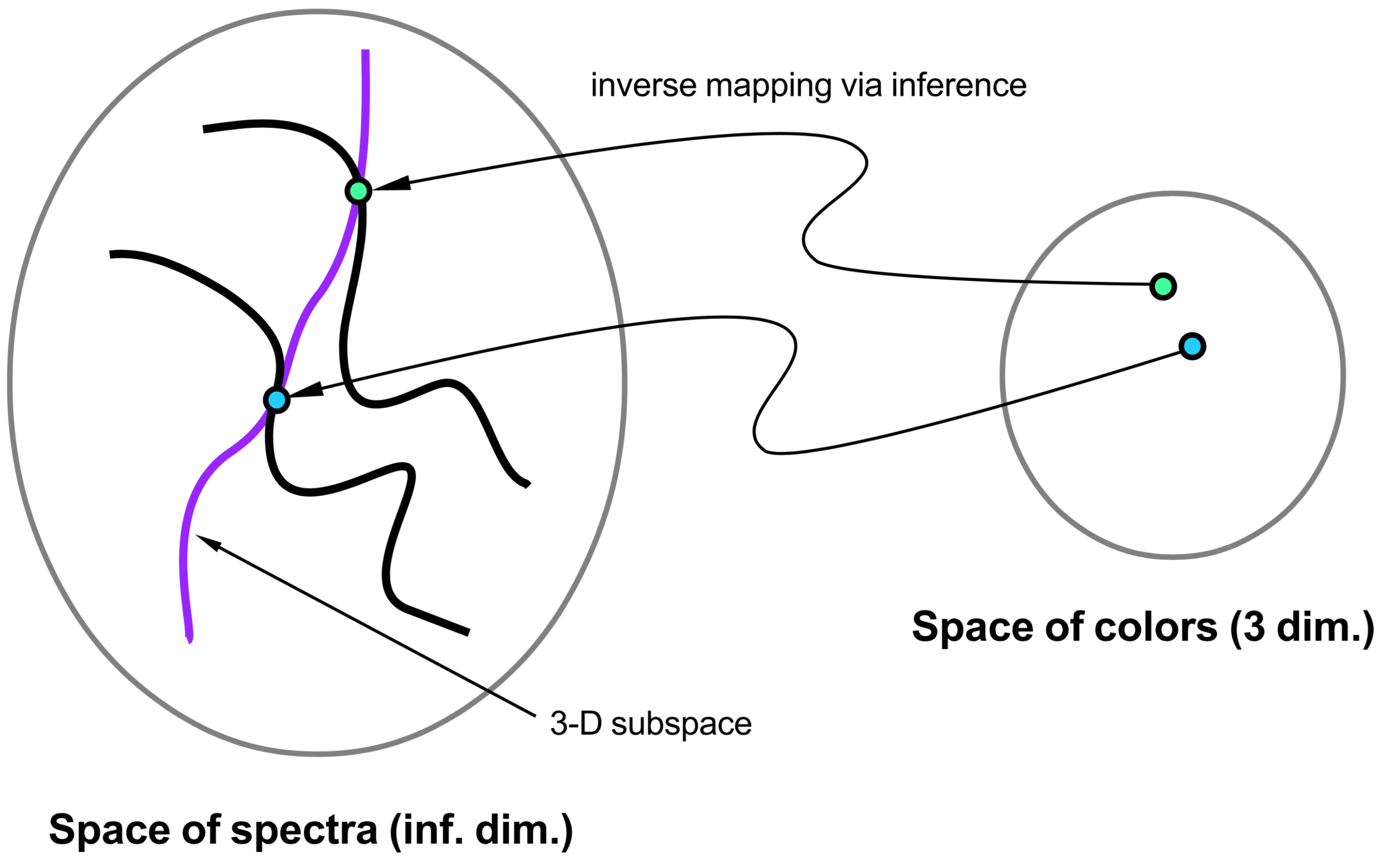
  we must invent the color product operator!

$$\mathbf{a} \otimes \mathbf{b}$$

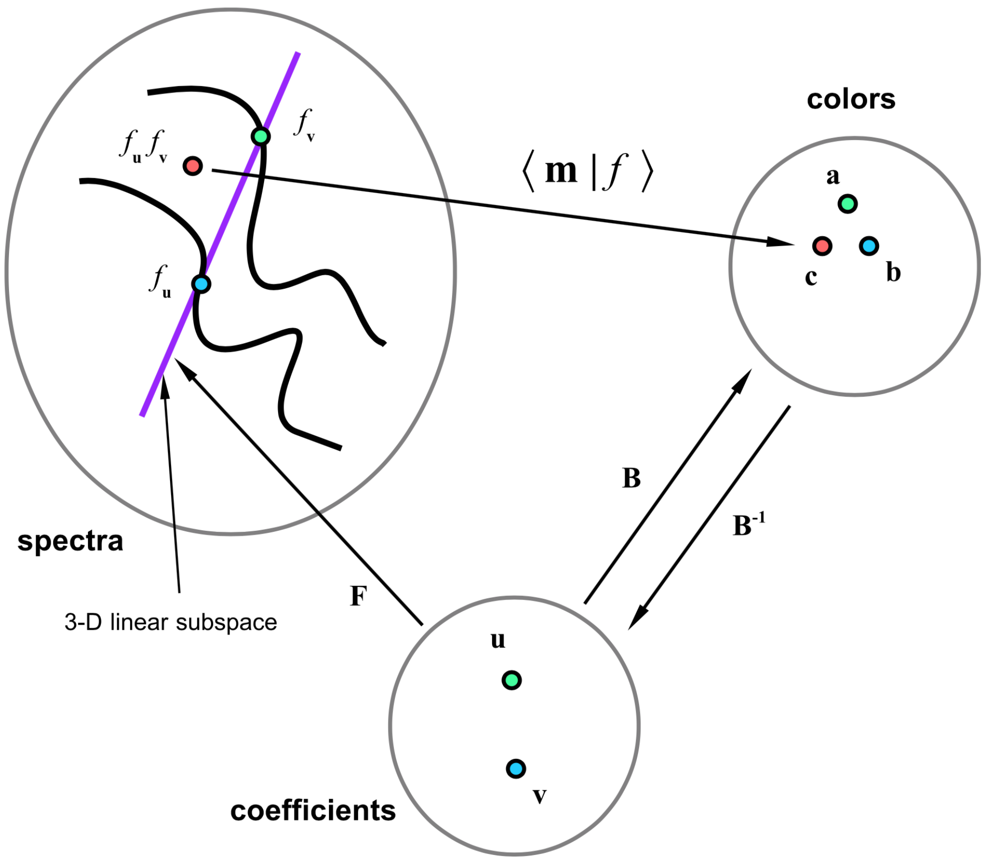
The ambiguity of metamers



General approach: spectral model



Example: linear spectral models



$$f_\mathbf{u} = \sum_{i=1}^{3} u^i f_i(\lambda)$$

History of linear models

- Sällström (1973)
- Brill (1978)
- Buchsbaum (1980)

History of linear models (cont.)

- Maloney and Wandell (1986)

Color constancy: a method for recovering surface spectral reflectance

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Received July 18, 1985; accepted August 9, 1985

Human and machine visual sensing is enhanced when surface properties of objects in scenes, including color, can be reliably estimated despite changes in the ambient lighting conditions. We describe a computational method for estimating surface spectral reflectance when the spectral power distribution of the ambient light is not known.

History of linear models (cont.)

- Maloney (1986)

Color constancy: a method for recovering surface spectral reflectance

Evaluation of linear models of surface spectral reflectance with small numbers of parameters

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Received March 10, 1986; accepted June 27, 1986

Recent computational models of color vision demonstrate that it is possible to achieve exact color constancy over a limited range of lights and surfaces described by linear models. The success of these computational models hinges on whether any sizable range of surface spectral reflectances can be described by a linear model with about three parameters. In the first part of this paper, I analyze two large sets of empirical surface spectral reflectances and examine three conjectures concerning constraints on surface reflectance: (1) that empirical surface reflectances fall within a linear model with a small number of parameters, (2) that empirical surface reflectances fall within a linear model composed of band-limited functions with a small number of parameters, and (3) that the shape of the spectral-sensitivity curves of human vision enhance the fit between empirical surface reflectances and a linear model. I conclude that the first and second conjectures hold for the two sets of spectral reflectances analyzed but that the number of parameters required to model the spectral reflectances is five to seven, not three. A reanalysis of the empirical data that takes human visual sensitivity into account gives more promising results. The linear models derived provide excellent fits to the data with as few as three or four parameters, confirming the third conjecture. The results suggest that constraints on possible surface-reflectance functions and the "filtering" properties of the shapes of the spectral-sensitivity curves of photoreceptors can both contribute to color constancy. In the last part of the paper I derive the relation between the number of photoreceptor classes present in vision and the "filtering" properties of each class. The results of this analysis reverse a conclusion reached by Barlow: the "filtering" properties of human photoreceptors are consistent with a trichromatic visual system that is color constant.

History of linear models (cont.)

- Jeff makes a foray in '91

Sixteenth Annual Interdisciplinary Conference
Teton Village, Jackson, Wyoming
January 20 - 25, 1991
Organizer: George Sperling, New York University

Proceedings

Wednesday, January 23, 4:00 - 8:00 p.m. **Perceptual Processes**
Bart Anderson, *Harvard*, Disambiguating Ambiguity in 3D. .\" Will mail his registration to NYU - v
David Brainard, *U. Rochester*, Sampling and Reconstruction in Human Spatial Vision.
.\" Cancelled, kid's illness: Steve Zucker, *McGill U.*, Contrast, Contours and Cytochrome Oxidase.
Jitendra Malik, *UC Berkeley*, Curvilinear Grouping and Texture Segregation.
.\" Jim Enns, *U. British Columbia*, Rapid Recovery of Three-Dimensional Structure in Early Vision.
Terry Boult, *Columbia*. Symmetry and skew symmetry.
Jeff Mulligan, *NASA-Ames* , A Model of Subtractive Color Mixture and Color Transparency.

History of linear models (cont.)

- D'Zmura (1992)

Color constancy: surface color from changing illumination

Michael D'Zmura

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Received September 5, 1991; revised manuscript received October 30, 1991; accepted November 13, 1991

Viewing the lights reflected by a set of three or more surfaces, a trichromatic visual system can recover three color-constant descriptors of reflectance per surface if the color of the surfaces' illuminant changes. This holds true for a broad range of models that relate photoreceptor, surface, and illuminant spectral properties. Changing illumination, which creates the problem of color constancy, affords its solution.

History of linear models (cont.)

- Iverson and D'Zmura (1994)

Color constancy: surface color from changing illumination

Color constancy. I. Basic theory of two-stage linear recovery of spectral descriptions for lights and surfaces

Color constancy. II. Results for two-stage linear recovery of spectral descriptions for lights and surfaces

Color constancy. III. General linear recovery of spectral descriptions for lights and surfaces

Criteria for color constancy in trichromatic bilinear models

Geoffrey Iverson and Michael D'Zmura

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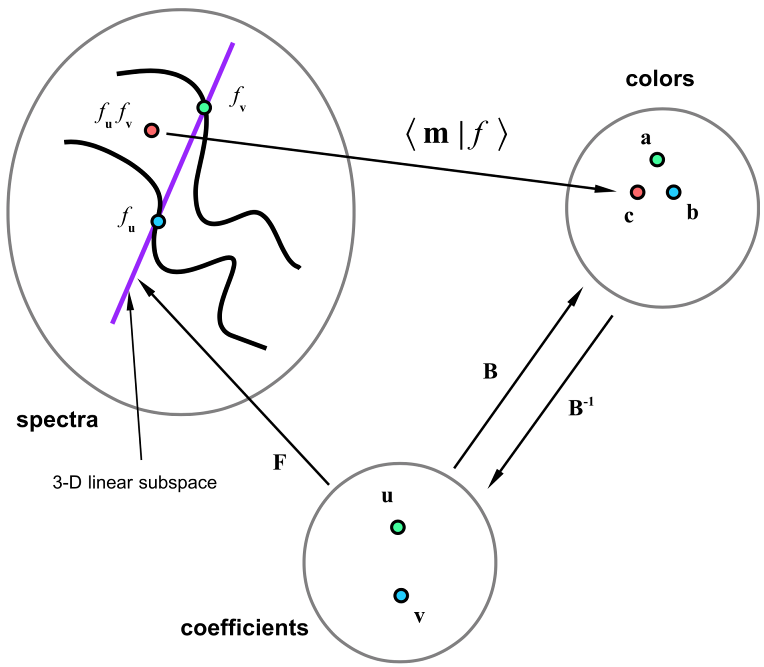
Received July 20, 1993; revised manuscript received January 26, 1994; accepted January 26, 1994

Bilinear color product



$$
\begin{aligned}
\mathbf{a} \otimes \mathbf{b} &= \mathbf{c} \\
&= \langle \mathbf{m} | f_{\mathbf{u}} f_{\mathbf{v}} \rangle \\
&= \mathbf{u}^{\intercal} \mathbf{P} \mathbf{v} \\
&= (\mathbf{B}^{-1}\mathbf{a})^{\intercal} \mathbf{P} \mathbf{B}^{-1} \mathbf{b} \\
&= \mathbf{a}^{\intercal} (\mathbf{B}^{-1})^{\intercal} \mathbf{P} \mathbf{B}^{-1} \mathbf{b}
\end{aligned}
$$

$$p_{ij}^k = \langle m^k | f_{ij} \rangle$$

Bilinear color division

$$\mathbf{c} = \mathbf{a} \otimes \mathbf{b} = \mathbf{T_a b}$$

$$\mathbf{T_a} = \mathbf{a}^\top (\mathbf{B}^{-1})^\top \mathbf{P} \mathbf{B}^{-1}$$

$$\mathbf{b} = \mathbf{T_a}^{-1} \mathbf{c}$$

$$\mathbf{b} = \mathbf{c} \oslash \mathbf{a}$$

- division = "discounting the illuminant"

Associativity

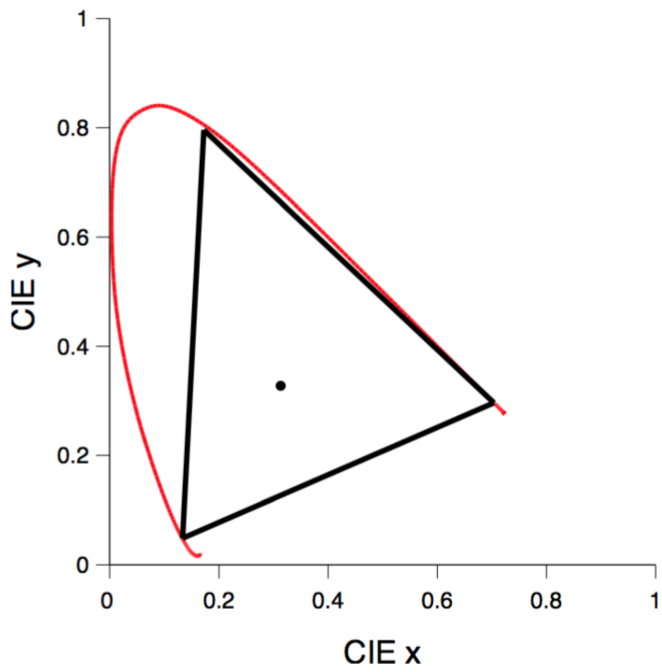
$$(\mathbf{a} \otimes \mathbf{b}) \otimes \mathbf{c} \stackrel{?}{=} \mathbf{a} \otimes (\mathbf{b} \otimes \mathbf{c})$$

- Associative law can fail for the linear model
- Approximation required when product spectra lie outside the model space
- Closure under multiplication guarantees associativity

A problem with the linear model

- Not all combinations are legal spectra
- Spectral values cannot be negative
- Choice of basis functions determines valid gamut

Example: RGB gamut



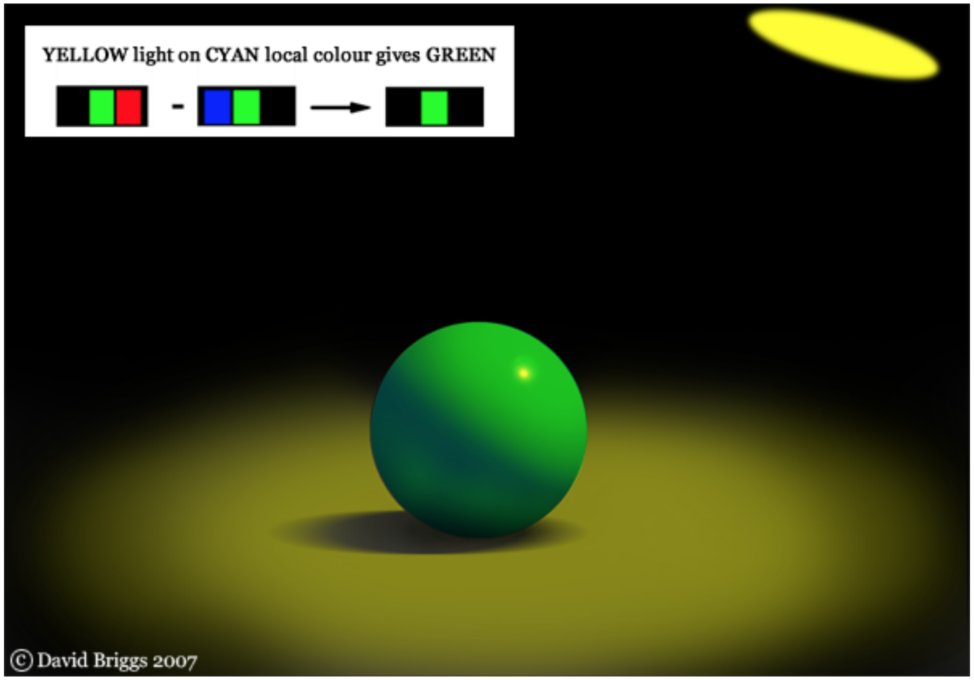
approximate ITU Rec. 2020

$$f_i(\lambda) = \begin{cases} \alpha_i & |\lambda - \lambda_i| \leq \Delta\lambda_i \\ 0 & \text{otherwise} \end{cases}$$

$$f_{ii}(\lambda) = \alpha_i f_i(\lambda),$$
$$f_{ij}(\lambda) = 0 \qquad i \neq j$$

$$\alpha_i = 1$$

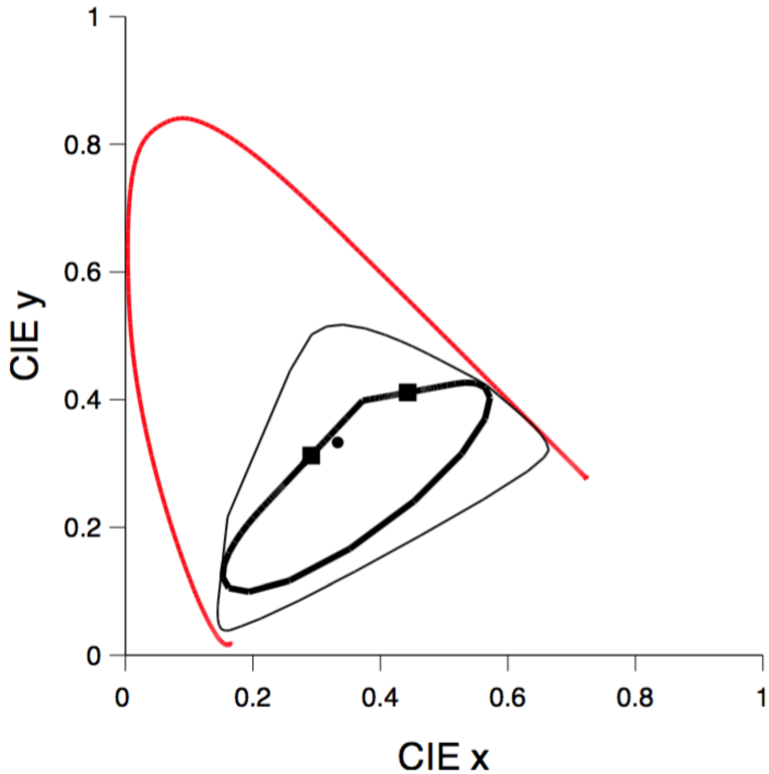
RGB Spectral Model

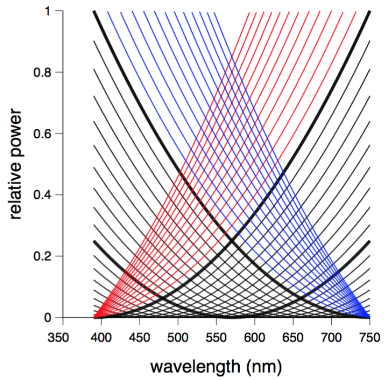


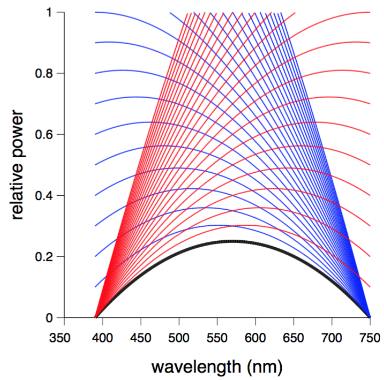
from http://www.huevaluechroma.com/051.php

- OpenGL spec. does not specify!

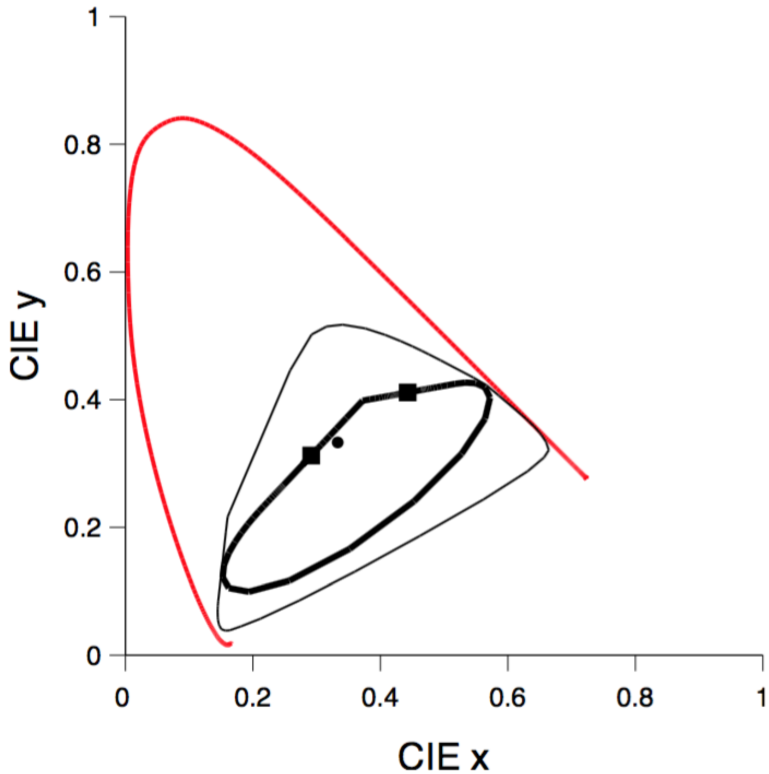
Example: Quadratic gamut

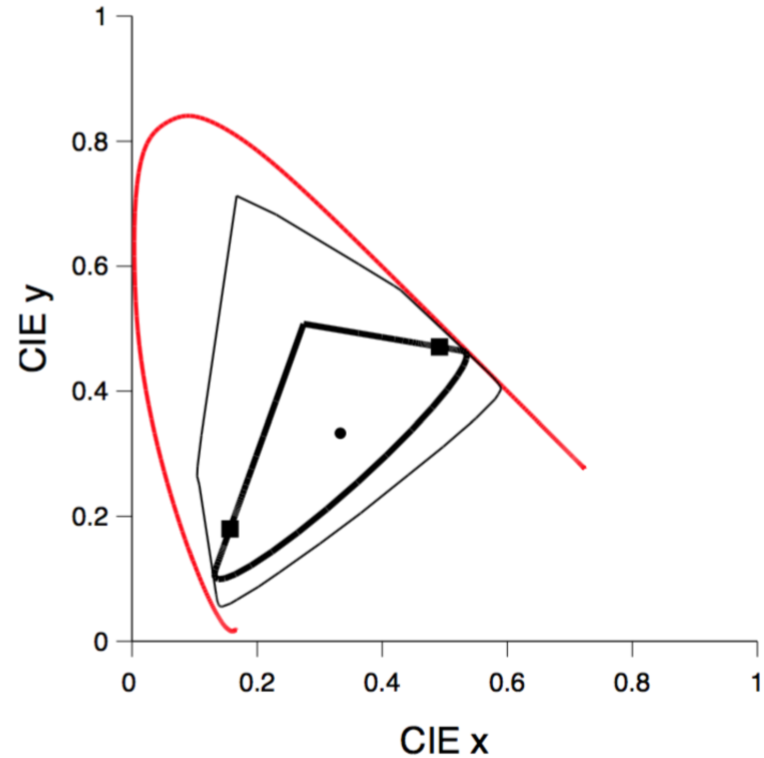




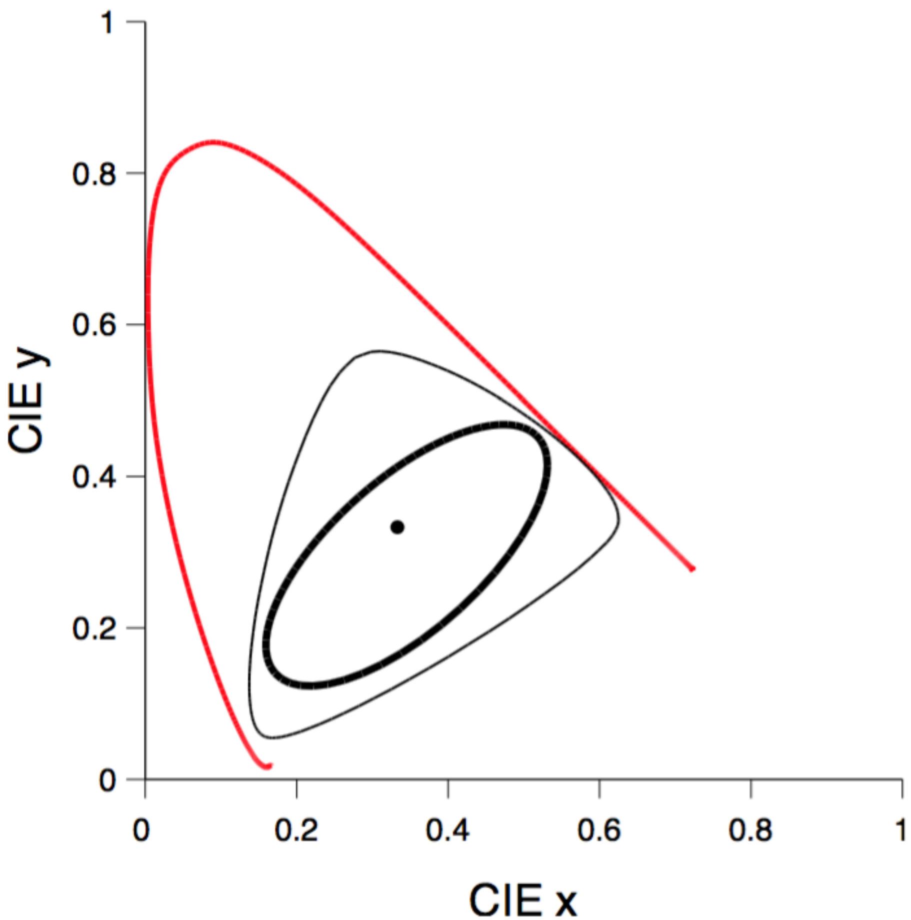


Example: Quadratic gamut





Example: Sinusoidal gamut

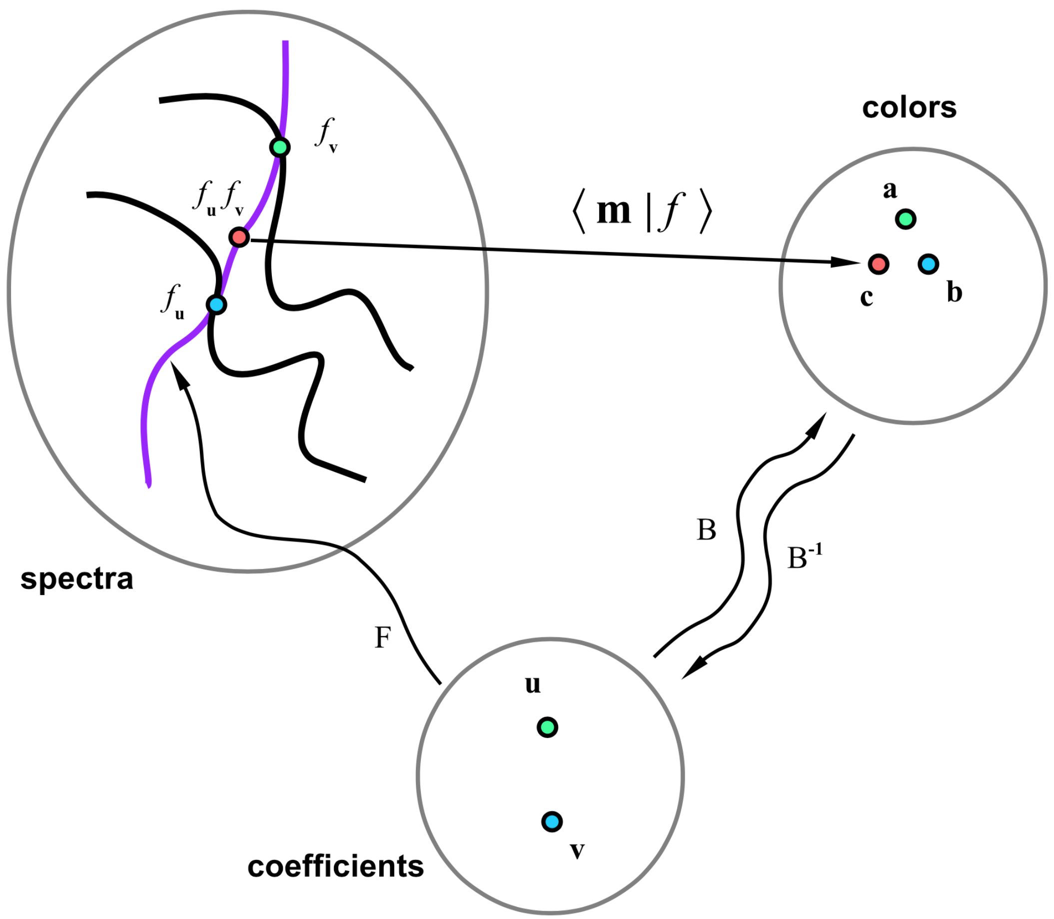


Log-linear spectral models

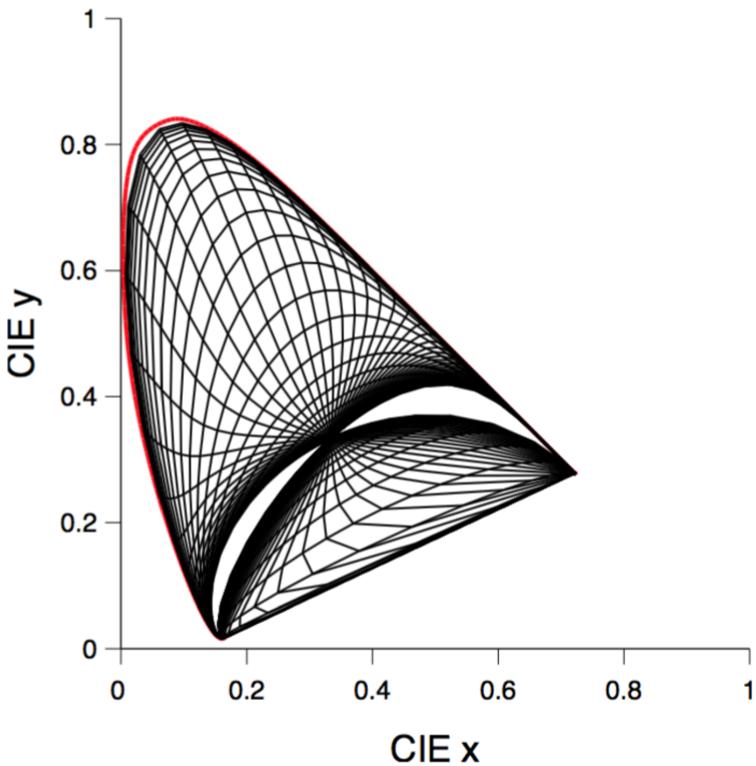
$$\log(f_{\mathbf{u}}) = \sum_{i=1}^{3} u^i f_i \qquad f_{\mathbf{u}} = \prod_{i=1}^{3} e^{u^i f_i}$$

- Suggested by Golz and MacLeod (2002), MacLeod and Golz (2003)
- Linear function space in log energy
- Closed under multiplication - associative law holds
- quadratic → Gaussian (and inverse-Gaussian)
- sinusoidal → Von Mises

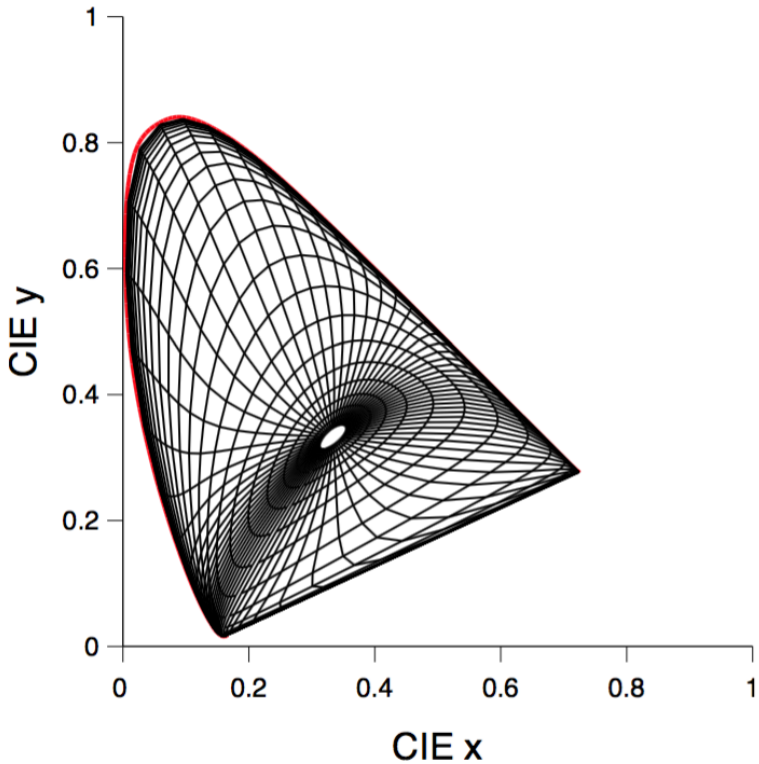
Log-linear spectral models (cont.)



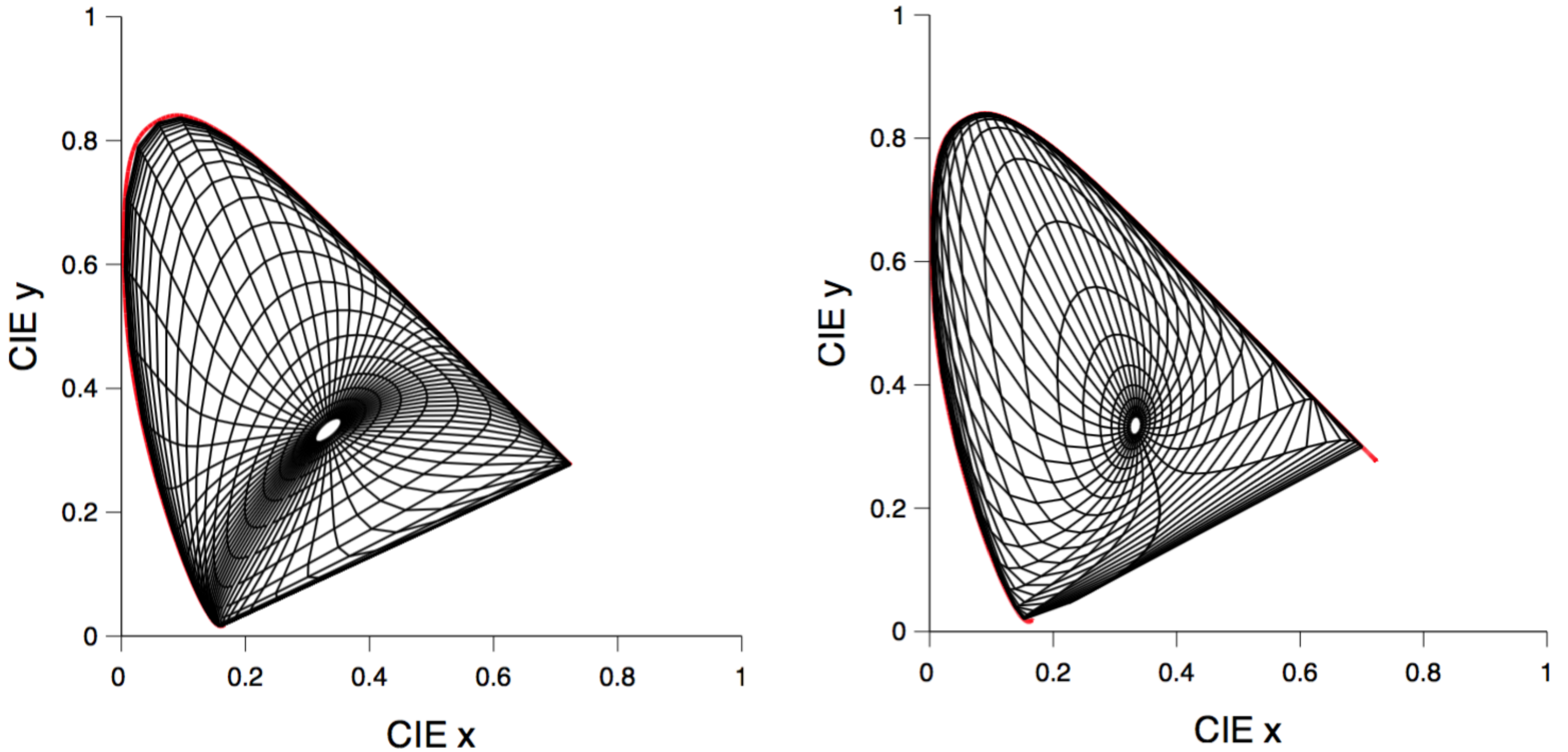
Example: Gaussian gamut



Example: Von Mises gamut



Example: Von Mises gamut



Advantages of the Von Mises model

- More physically plausible than RGB
- Can represent all colors
- Nonlinear wavelength transformation can generate individual differences (Abney effect, unique hue settings)
- Computational issues not solved - neural network?

Summary

- A computational framework for an algebra of colors to predict additive and subtractive color mixture
- Applications to graphics and perception
- Log-linear spectral models provide best performance